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DETERMINATION OF THE CHARACTERISTIC
IMPEDANCE OF UNIFORM
TRANSMISSION LINES

by

H.N. Dawirs and W.S. Tulloss

Boeing Airplane Company
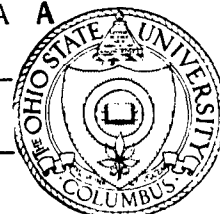Seattle 24, Washington

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Department of ELECTRICAL ENGINEERING

THE OHIO STATE UNIVERSITY
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Columbus 8, Ohio

REPORT

by

THE OHIO STATE UNIVERSITY RESEARCH FOUNDATION
COLUMBUS 8, OHIO

Cooperator   Boeing Airplane Company
Seattle 24, Washington

Investigation of   Specialized Microwave Circuit Design

Subject of Report   Determination of the Characteristic
Impedance of Uniform Transmission Lines

Submitted by   H.N. Dawirs and W.S. Tulloss
Antenna Laboratory
Department of Electrical Engineering

Date   31 July 1959

ABSTRACT

In order to conduct the investigation of small discontinuities in transmission lines of various cross sections it is necessary to know the characteristic impedances of such lines. A number of experimental methods of determining the characteristic impedances of transmission lines of arbitrary cross section were considered and are described in this report. At the same time,the literature was surveyed for analytical methods, a number of which were found for slab line and various forms of strip type transmission lines. These are listed in the References. As a check, several different slab line impedances were determined by the experimental methods and compared to a calculated curve.

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DETERMINATION OF THE CHARACTERISTIC
IMPEDANCE OF UNIFORM TRANSMISSION LINES

I. INTRODUCTION

In the study of strip line discontinuities it is necessary to know the characteristic impedance of the strip lines involved. The characteristic impedance of slab line (round center conductor between two parallel plane outer conductors) is also required since it is convenient for use as a transition section between the strip line and the coaxial slotted line used for measurement. This report is a review of experimental methods used to determine these characteristic impedances. These methods are general in that they can be used to determine characteristic impedances of uniform transmission lines of any cross section.

In the meantime a review of the literature revealed a number of formulas and curves for determining the characteristic impedances of strip line and slab line. References to these are listed at the end of this report.

II. EXPERIMENTAL PROCEDURES

A. Simple Shorted Stub

In theory at least it is necessary only to measure the input impedance Z_i of a shorted stub constructed from a known length of the transmission line under consideration in order to be able to determine its characteristic impedance. The input impedance to this stub must be

(1) $$Z_i = j\, Z_c \tan \beta l$$

where

Z_i is the known input impedance

Z_c is the characteristic impedance of the transmission line from which the stub is constructed and whose value is to be determined.

l is the (known) length of the stub.

It is quite simple to calculate the value of the characteristic impedance Z_c from the known values of Z_i and l.

B. Open and Short Circuit Termination

Another method, simple in theory, of determining the characteristic impedance of a symmetrical network, which in our case is a section of transmission line, is the well known open and short circuit method. That is, the input impedance of the network is measured, first, with the output terminals open to determine the open circuit impedance Z_{OC}, and, then, with the output terminals short circuited to determine the short circuit impedance Z_{SC}. The characteristic impedance of the network is, then, the geometric mean of the measured values of the open and short circuit impedances: i.e.,

$$(2) \qquad Z_C = \sqrt{Z_{OC} Z_{SC}} \ .$$

The length of the section of line in this case is immaterial as long as it is not an integral number of wavelengths, in which case (2) would become indeterminate.

C. Filter Analysis Chart

The O.S.U. Filter Analysis Chart (see Figs. 5 and 6) consists of a set of coordinates designed to be superimposed on a Smith Chart for the specific purpose of determining characteristic impedances and propagation constants of microwave networks, from input impedance measurements.

The network under consideration is terminated in any real load impedance, $Z_0 = R_0$. The input impedance is then normalized to the real terminating impedance R_0, and plotted on an ordinary Smith Chart. When the point is transferred to the filter analysis chart it will lie on a characteristic impedance coordinate which is the characteristic impedance of the transmission line being measured, normalized to the terminating impedance. Z_0.

Here again the length of transmission line must be other than an integral number of half-wavelengths long for the characteristic impedance to be determinate, otherwise the length is arbitrary.

III. EXPERIMENTAL PROCEDURE INCLUDING THE EFFECT OF JUNCTIONS

While any of the above procedures may appear quite simple and straightforward, a practical difficulty arises as soon as measurements are attempted. The transition from the slotted line being used to make the measurement to the transmission line being measured introduces a disturbance into the measurements since tney have different cross sections. Thus, the slotted line measurements always include the disturbance of the junction.

If only an abrupt step discontinuity at the junction of the two dissimilar lines is involved, the effect is principally that of a shunt capacitor at the point of discontinuity.[1,2] This is not exactly true since the disturbance is actually distributed in the vicinity of the discontinuity due to the distortion of the field in that region, although the discrepancy may be slight if the extent of the disturbance is small in terms of a wavelength.

A. Using Simple Shorted Stubs

If the disturbance can be considered to be a shunt admittance at the transition, as shown in Fig. 1, its value can be determined by measuring
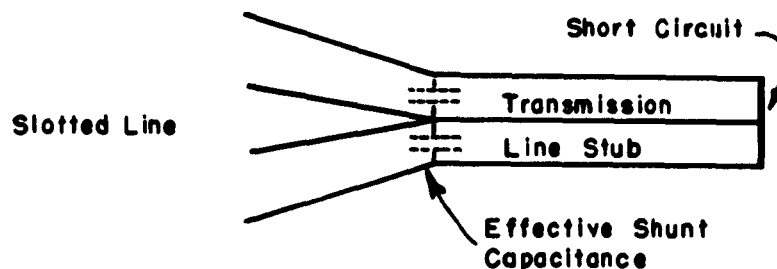

Fig. 1. Effective shunt capacity due to discontinuity at the junction of the transmission line under consideration and the slotted line used for measurement.

the input admittance to a stub which is a quarter of a wavelength long. Since the input admittance of the quarter wavelength stub is zero, only the shunt admittance due to the discontinuity is measured. With the shunt admittance of the transition so established, it may be subtracted from the input admittance measurements of other stub lengths to determine the true input admittance to the shorted stubs from which to calculate the characteristic admittance of the transmission line as before.

In order to obtain increased accuracy, it is often necessary to average a number of independent measurements by means of a curve. Such a curve for our purpose may be obtained by taking measurements as a function of the length of the shorted stub. A plot of the resulting impedances is not convenient for an averaging curve over a very wide range because it periodically goes to infinity. A more desirable curve for averaging is obtained by plotting the shift of the null on the slotted line as a function of the length of the short circuited stub.

This curve will be a straight line with a slope of one if there is no shunt susceptance and the characteristic admittance of the stub is equal to that of the slotted line.

If the characteristic admittance of the stub is different from that of the slotted line and there is no shunt susceptance, the curve oscillates about the 45° line, crossing it when the stub is any multiple of a quarter wavelength long.

If, in addition to the characteristic admittances of the stub and slotted line being different, there is a shunt susceptance, the curve still oscillates about the 45° line, crossing it at multiples of a half wavelength, but with the alternate crossings shifted from the quarter wavelength points by the effects of the shunt susceptance.

Thus the value of the shunt capacity can be calculated from the departure of the curve from the 45° line at the odd quarter wavelength points, and the characteristic impedance of the stub transmission line can be calculated from the departure at any other point (with the exception of the half wave length points) by correcting for the shunt capacity.

Since only the departure of the curve from the 45° line is used, a more convenient and accurate curve may be obtained by plotting only this departure.

It may be difficult to obtain a number of different stub lengths for measurement to obtain a curve, with transmission lines of odd cross section. In these cases effective changes in stub length may be obtained by varying the frequency, if the shunt admittance is some known function of the frequency. If the shunt admittance is effectively a capacitance, the shunt admittance will be a linear function of the frequency which can be taken into account in the calculations. Figure 2 shows a typical plot of departure vs effective stub length for the determination of shunt susceptance of the discontinuity and the characteristic impedance of the transmission line, where the "departure" is the difference between the shift of the null in the slotted

line (from a short circuit reference position) caused by a short circuited stub, and the length of the stub itself. In this case the effective variation in stub length was achieved by varying the frequency.
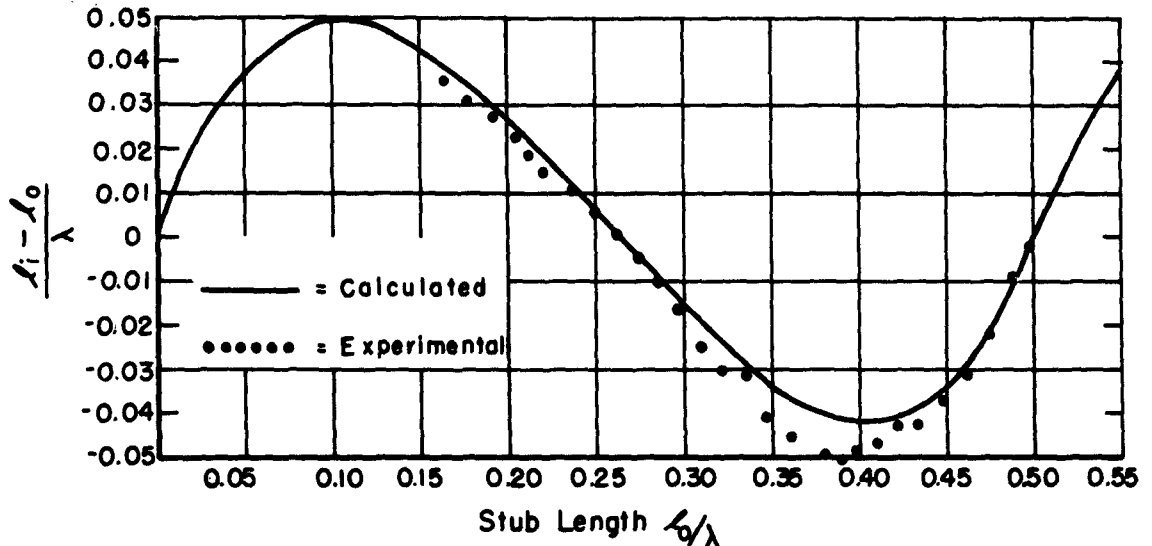

Fig. 2. Plot of difference between the shift of the null in a slotted line and the length of a shorted stub terminating the line as a function of the effective length of the shorted stub itself.

The plotted points are the results of experimental measurements from which values of characteristic impedance and shunt capacity were calculated. These values were then used to calculate a curve which was then drawn in (on Fig. 2) for comparison. There is some discrepancy between the two which perhaps could be reduced by further calculations. However, there are no doubt other disturbances, such as line inaccuracy, the fact that the junction disturbance is not strictly a shunt capacity at a point, etc., which are responsible for some of this displacement.

The characteristic impedance may also be determined directly from the slope of the curve at the half-wave crossover point as

(3) $Z_c = Z_i(1 + m)$

where

Z_c is the characteristic impedance of the stub

Z_i is the characteristic impedance of the slotted line

m is the slope of the curve at the half wave crossover point.

This formula is not only convenient but is quite reliable since, from a good curve, the slope can be determined quite well and the shunt capacity should have little effect in this region due to the extremely large shunt admittances of the stub. The formula may be developed as follows.

Consider the slotted line to be terminated in a shorted stub whose length is nearly a half wavelength long, say, $l_o = \lambda/2 + \Delta l_o$ where Δl_o is small. Then the shift on the slotted line will be nearly a half wavelength also; that is $l_i = \lambda/2 + \Delta l_i$ where Δl_i is small. Now the input admittance to the stub as measured by the slotted line will be

$$(4) \qquad -jY_i \cot \beta l_i = -jY_o \cot \beta l_o + jB_o$$

where

Y_i is the characteristic admittance of the slotted line

Y_o is the characteristic admittance of the stub

B_o is the shunt admittance at the junction of the stub with the slotted line.

Since l_i and l_o are both near a half-wavelength long, both cotangent functions will be large and, hence, B_o may be neglected in this region to obtain:

$$(5) \qquad Y_i \cot \beta l_i \cong Y_o \cot \beta l_o$$

or

$$(6a) \qquad \frac{Y_o}{Y_i} \cong \frac{\cot \beta l_i}{\cot \beta l_o}$$

$$(6b) \qquad = \frac{\tan \beta l_o}{\tan \beta l_i}$$

$$(6c) \qquad = \frac{\tan \beta (\lambda/2 + \Delta l_o)}{\tan \beta (\lambda/2 + \Delta l_i)}$$

$$(6d) \qquad = \frac{\tan \beta \Delta l_o}{\tan \beta \Delta l_i}$$

(6e) $\qquad \cong \dfrac{\Delta\ell_o}{\Delta\ell_i} = \dfrac{Z_i}{Z_o}$ for $\Delta\ell_o$ small.

Now when the differences are plotted, as in Fig. 2, the slope of the curve at the half wave crossover point is

(7) $\qquad m \cong \dfrac{(\lambda/2 + \Delta\ell_i) - (\lambda/2 + \Delta\ell_o)}{\Delta\ell_o} = \dfrac{\Delta\ell_i}{\Delta\ell_o} - 1$

so that

(8) $\qquad Z_o = Z_i \dfrac{\Delta\ell_i}{\Delta\ell_o} = Z_i(m + 1)$

as indicated previously.

Figure 3 shows the characteristic impedances and Fig. 4 shows the shunt capacitances at the junction as determined from the measurements.

B. Filter Analysis Chart

The filter analysis chart method may also be used to determine the characteristic impedances of transmission lines through the small step discontinuities at the junction of the line with the slotted measuring line. If the real load used to terminate the test section of transmission line is constructed to terminate the slotted line without a discontinuity, then, when it terminates the test section, the discontinuity at the junction of the load with the test section at one end will be identical to the discontinuity of the test section with the slotted line at the other. Thus, the input impedances to the test section, measured as a function of frequency, will not lie on a perfect circle, as described previously, due to the disturbance of the two discontinuities. However, when the test section is a quarter of a wavelength long (or some odd multiple), the two identical disturbances tend to cancel giving reliable measurements in this region. Fortunately this is the neighborhood in which the most reliable results are usually obtained any way. Thus, in determining the characteristic impedance of a section of unknown transmission line by means of a filter analysis chart method, impedance measurements need be taken only in the region where the test section is a quarter of a wavelength long.

Examples of such a procedure are shown in Fig. 5 for a round center conductor in a rectangular outer conductor and in Fig. 6 for both the center conductor and the outer conductor, rectangular in cross section.
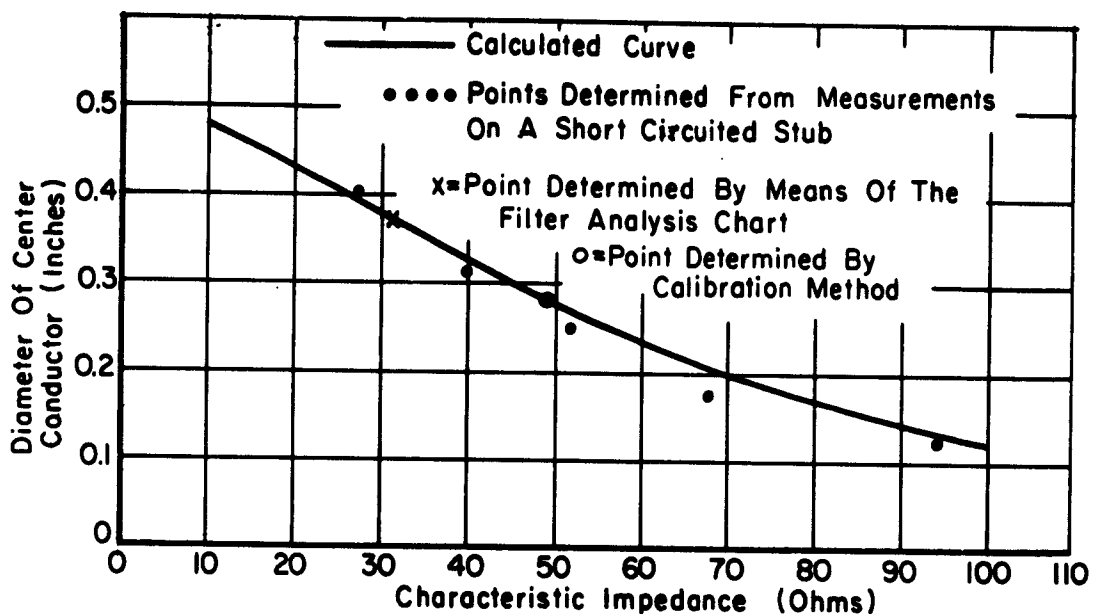
Fig. 3. Plot of the characteristic impedances of slab line
consisting of a round center conductor in a 1/2"
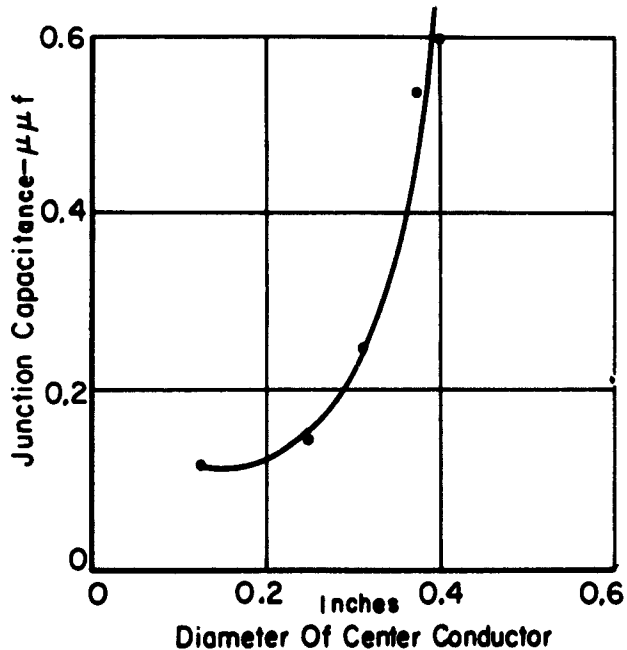x 1 1/8" rectangular outer conductor.

Fig. 4. Plot of the equivalent shunt capacity at the junction
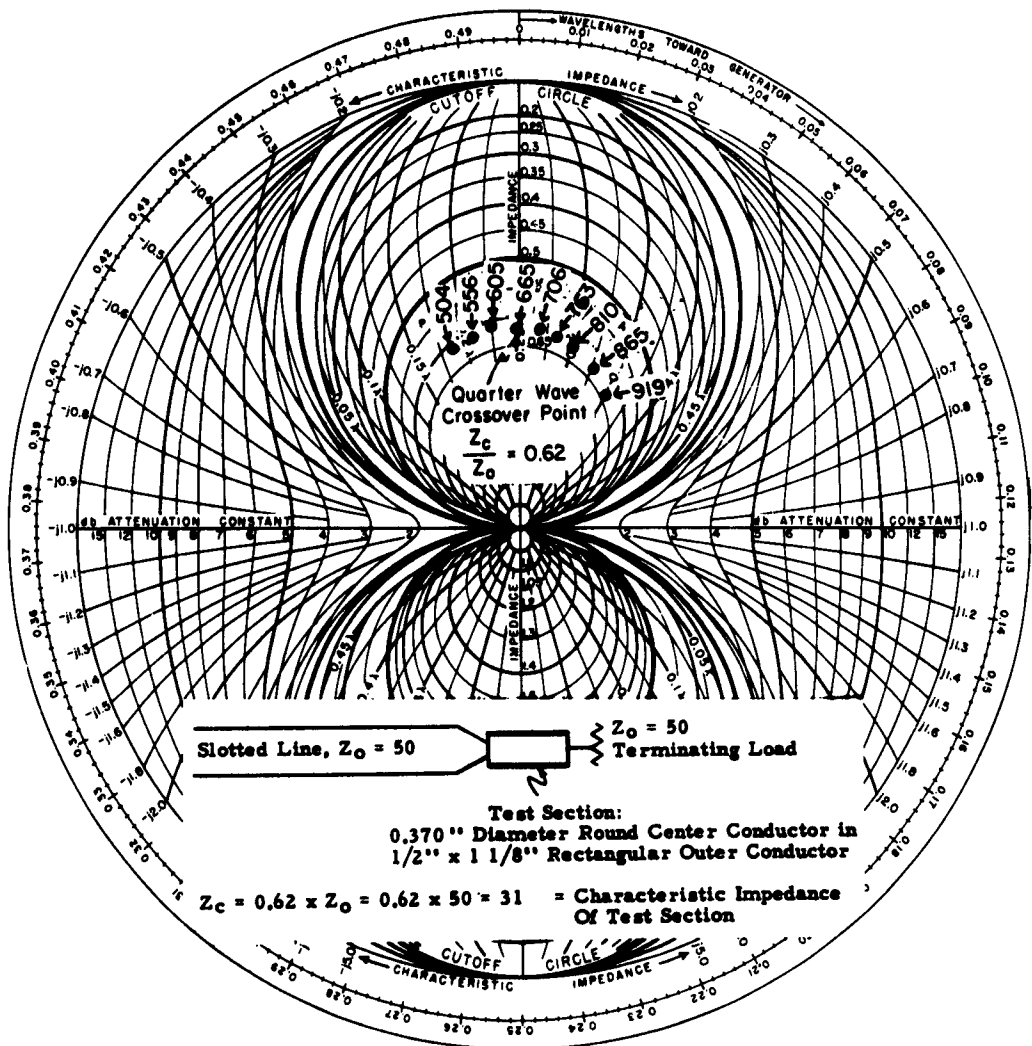of the slab line with the coaxial slotted line.

Fig. 5. Locus of input impedances of the slab line test section
as transferred from a Smith Chart plot normalized to
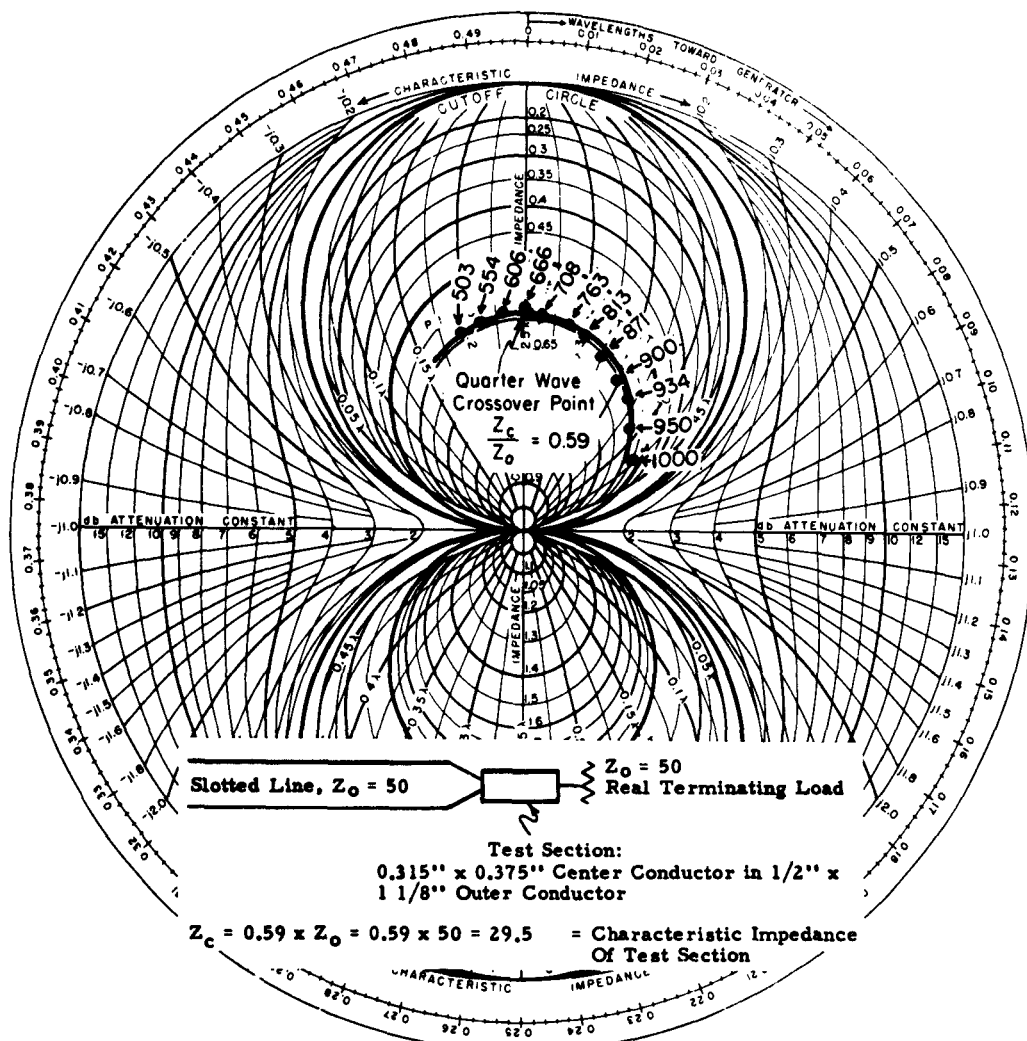the terminating resistance.

Fig. 6. Locus of input impedances of a strip line test section
as transferred from a Smith Chart plot.

The characteristic impedance of 31Ω determined from Fig. 5 for the round center conductor is plotted on Fig. 3 for comparison with values determined by the previous method. A curve is also plotted according to calculations using a formula due to Wheeler[3, 4] for comparison.

C. Calibration Method

If it is convenient to vary the length of the shorted stub in a known manner, a more accurate calibration procedure can be used to determine the characteristic impedance (or admittance) of the transmission line of which the stub is constructed. This method can yield very accurate results if the stub lengths are accurately known and the only disturbance is the shunt susceptance at the junction of the stub with the slotted line (see Fig. 7). A short circuit reference point is first determined by locating a null in the slotted section with the slotted line terminated in a short circuit. Now let ℓ_i be the distance from this short circuit reference null to the null obtained with the slotted line terminated in a stub of length ℓ_o. The input admittance of the stub alone will be

$$(9) \qquad Y_o = -j\, Y_o\, \cot\, \beta\ell_o$$

where Y_o is the characteristic admittance of the stub, which is being determined. The impedance measured by the slotted line is

$$(10) \qquad Y_i = -j\, Y_i\, \cot\, \beta\ell_i = -j\, Y_o\, \cot\, \beta\ell_o + j\, B_o$$

which includes the shunt susceptance B_o of the junction as well as the input admittance of the stub. (Y_i is the characteristic admittance of the slotted line.) This equation can be put into the form

$$(11) \qquad \cot\, \beta\ell_i = \frac{Y_o}{Y_i}\, \cot\, \beta\ell_o - \frac{B_o}{Y_i}$$

which is a linear equation in $\cot\, \beta\ell_i$ and $\cot\, \beta\ell_o$ of the form

$$(12) \qquad y = mx + b$$

where

$y = \cot\, \beta\ell_i$ is the dependent variable

$x = \cot\, \beta\ell_o$ is the independent variable

$$m = \frac{Y_o}{Y_i} \quad \text{is the slope; and,}$$

$$b = -\frac{B_o}{Y_i} \quad \text{is the y-intercept.}$$

Thus the characteristic impedance Z_o os the transmission line of which the stub is constructed can be determined in terms of the known value of the characteristic impedance Z_i of the slotted line and the measured value of m as
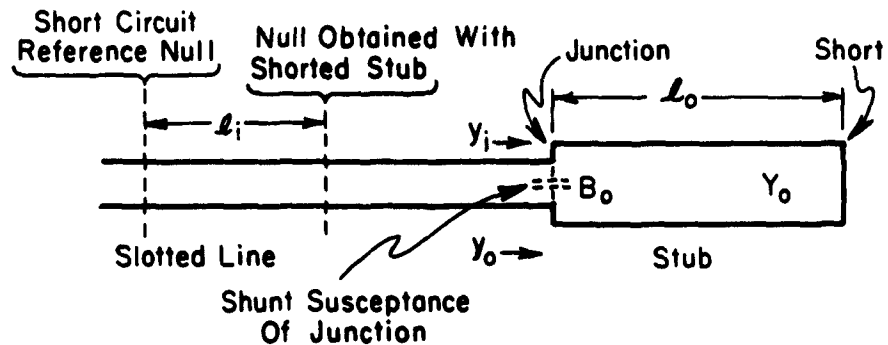
$$Z_o = Z_i/m.$$

Fig. 7. Circuit configuration for the calibration method of determining the characteristic impedance of a transmission line.

Since only the positions of sharp nulls are required in the measurements, the values of ℓ_i can be determined very accurately. The only other requirement for accurate results is precision in determining the stub length and accuracy in the calculations. A number of points may be determined (corresponding to different stub lengths) and plotted for greater reliability. The slope may then be taken directly from the graph.

Figure 8 shows such a plot for a 0.14" diameter round center conductor in a 1/4" x 1" rectangular outer conductor. The slope is about 1.03 so that the characteristic impedance is about $Z_c = 50 \div 1.03 = 49\,\Omega$. This impedance (with the cross section dimensions scaled up so that the plate separation is 0.5" to conform with the scale of Fig. 3) is plotted on Fig. 3 as a comparison with the calculated curve and the other measured values.

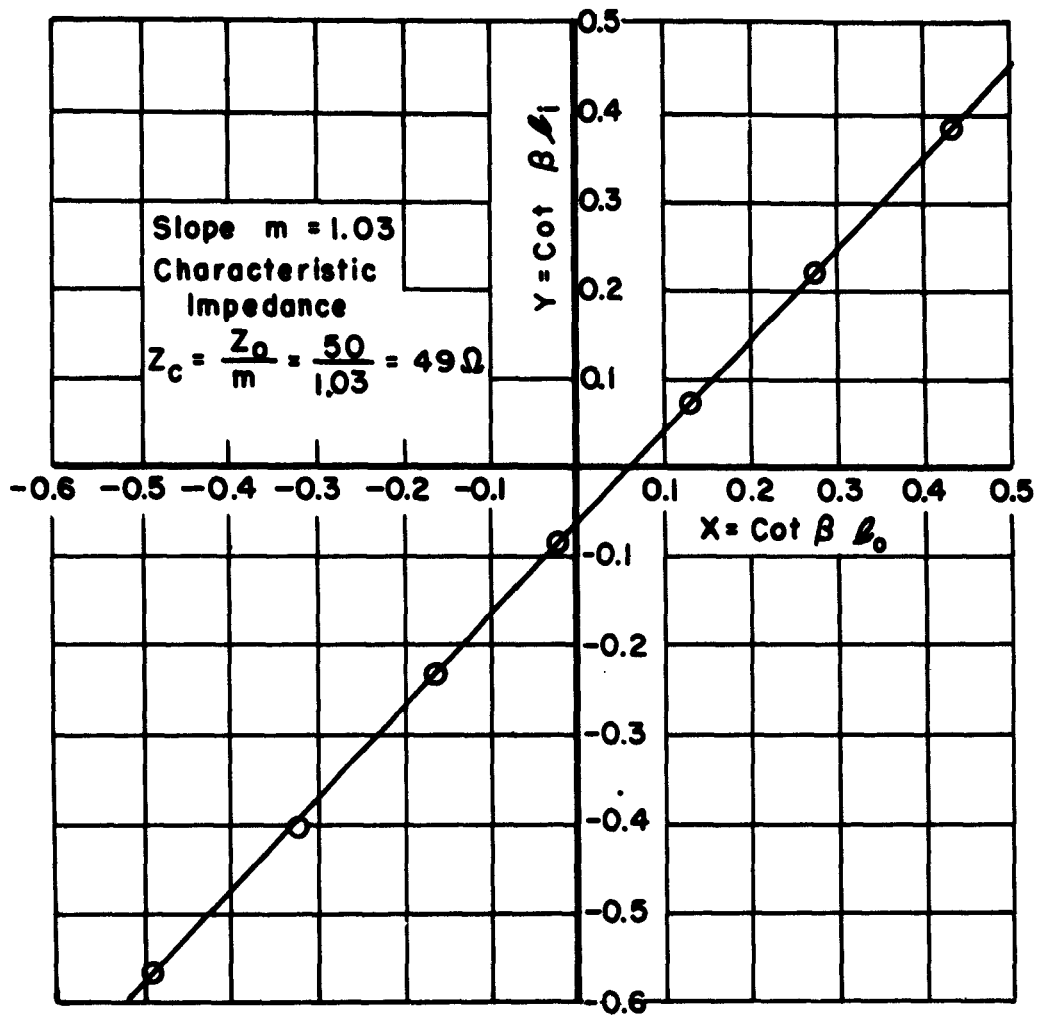This procedure may be somewhat more difficult than the others but has a greater capability for accuracy.

Fig. 8. Calibration curve of 0.14" round center conductor in
a 1/4" x 1" rectangular outer conductor.

IV. CONCLUSIONS

All of the procedures described rely on an accurate slotted line. If there are other disturbances due to transitions of various kinds between the junction and the slotted section, these will affect the results and must be taken into account if they limit the accuracy desired. These transitions can be calibrated by procedures described in a previous report.

The measurements of slab-line characteristic impedances by the various methods agree quite well with the calculated curve as shown in Fig. 3

Some discrepency may be expected due to the fact that the curve assumes infinite parallel plane outer conductors while the measurements were made with a closed rectangular outer conductor to prevent higher order modes and radiation. The sides are far enough out however so that they should not have much effect, especially at the lower impedances.

REFERENCES

1.   Whinnery, J.R. and Jamieson, H.W., "Equivalent Circuits for Discontinuities in Transmission Lines," Proc. I.R.E., Vol. 32, pp. 98-115, February, 1944.

2.   Whinnery, J.R., Jamieson, H.W., and Robbins, T.E., "Coaxial-Line Discontinuities," Proc. I.R.E., Vol. 32, pp. 695-709, November 1944.

3.   Wheeler, H.A., "The Transmission-Line Properties of a Round Wire Between Parallel Plates," I.R.E. Trans. on Antennas and Propagation, Vol. AP-3, pp. 203-207, October 1955.

4.   Bad.vannis, G.M., "The Power Handling Capacity of Slab Lines," 1958 I.R.E. WESCON Convention Record, Part 1, Antennas and Propagation, Microwave Theory and Techniques, pp. 35-38.

5.   Whaley W.B. and Eldred, W.N., "A New Type of Slotted Line Section," Proc. I.R.E., Vol. 38, pp. 244-288, March 1950.

6.   Cohn, Seymour B., "Characteristic Impedance of the Shielded-Strip Transmission Line," Trans. I.R.E., Vol. MTT-2, No. 2, pp. 52-57, July, 1954.

7. Cohn, Seymour B., "Problems in Strip Transmission Lines," _Trans. I.R.E._, Vol. MTT-3, No. 2, pp. 119-126, March, 1955.

8. Begovich, Nicholas A., "Capacity and Characteristic Impedance of Strip Transmission Lines with Rectangular Inner Conductors," _Trans. I.R.E._, Vol. MTT-3, No. 2, pp. 127-133, March, 1955.

9. Pease, Robert L. and Mingins, Charles R., "A Universal Approximate Formula for Characteristic Impedance of Strip Transmission Lines with Rectangular Inner Conductors," _Trans. I.R.E._, Vol. MTT-3, No. 2, pp. 144-148, March, 1955.

10. Dawirs, H.N., Boehnker, C.H., and Garbacz, R.J., "A Simple Slotted-Line Method of Accurately Measuring Impedances through an Unknown Transition Section," Report 762-3, 31 December 1957, Antenna Laboratory, The Ohio State University Research Foundation; prepared for the Boeing Airplane Co., Seattle 24, Washington.